TRANSACTIONS OF THE OPTICAL SOCIETY

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LIST OF HONORARY FELLOWS AND FELLOWS

AT the Meeting held on 14th November, 1918, the President announced that the Council had elected the following as Honorary Fellows and Fellows of the Society. This forms the first list of Fellows under the new Rules.

* *Names of Fellows who have filled the Office of President of the Society.*
† *Names of Fellows who have died since accepting Fellowship.*

HONORARY FELLOWS

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SOME GENERALISED FORMS OF AN OPTICAL EQUATION

By T. SMITH, B.A.

(From The National Physical Laboratory)

Read and discussed, 14th November, 1918

SUMMARY. The paraxial equation for refraction at a spherical surface connecting conjugate points on the axis may be made an exact relation for all rays by the inclusion of an additional factor. All the angles required to specify a ray may be expressed in terms of three angles, some one of which vanishes when the rays pass through any one of the three pairs of conjugate points at which there is freedom from aberration. The additional factor to be introduced is the product of the tangents of the halves of any two of these three angles, and three exact equations of refraction are thus found. The correcting factor has only to be known approximately to enable the refraction of the ray to be determined to a high order of accuracy. When rays are to be traced through a series of surfaces the most convenient form of the equation at each surface is

$$\frac{\mu'}{x'-p}-\frac{\mu}{x-p}=\frac{\mu'-\mu}{r-p},$$

where p may be regarded as a correction to the radius of curvature and as a displacement of the effective unit point.

The general theory of image formation in optical instruments is based upon the well-known equation

$$\frac{\mu'}{x'}-\frac{\mu}{x}=\frac{\mu'-\mu}{r}=(\mu'-\mu)\,R=\kappa \qquad \ldots.(1),$$

or upon the two equivalent equations

$$\left.\begin{aligned} x\kappa &= -\mu\,(1-1/m)\\ x'\kappa &= \mu'\,(1-m)\end{aligned}\right\} \qquad \ldots.(2),$$

where m is the transverse linear magnification. These equations apply strictly only to paraxial rays. When it is necessary to obtain exact information about rays at finite inclinations to the axis the only course to adopt is to trace rays independently through the system of refracting surfaces. However such ray tracing is carried out the result obtained is equivalent to replacing κ by

$$K=(\mu'\cos\phi'-\mu\cos\phi)\,R,$$

where ϕ and ϕ' are the angles of incidence and refraction, and measuring all lengths along the ray instead of along the axis. In spite of the simplicity of this process it is not easy to predict with great accuracy the magnitude of the aberrations present in the system. The object of the present note is to point out how the

above equations may be modified so as to be applicable to all rays while retaining measurements along the axis. This modification does not lead to an exact method of ray tracing, but it enables the departure from the paraxial law at any surface to be readily estimated to a considerable order of accuracy, and under suitable conditions will give final coordinates for a ray as closely as can reasonably be desired. It should prove particularly convenient in studying eyepieces and other systems in which the obliquity of a ray to the axis is frequently so large that the usual paraxial formulae are almost useless.

The incident and refracted rays are determined by the distances x and x' of their intersections with the axis from the vertex of the refracting surface, and by their inclinations ψ and ψ' with the axis. If α is the angular aperture at which refraction takes place

$$\alpha = \phi - \psi = \phi' - \psi',$$

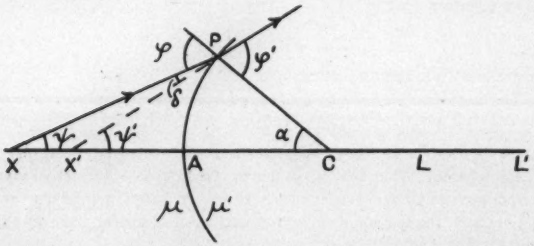
and if δ is the deviation

$$\delta = \phi' - \phi = \psi' - \psi.$$

These equations show that the angles $\phi' + \psi$ and $\phi + \psi'$ are equal. Denote them by γ*. The four angles ϕ, ψ, ϕ', ψ' are then expressed in terms of the three angles α, γ, δ by the equations

$$2\psi = \gamma - \alpha - \delta, \qquad 2\phi = \gamma + \alpha - \delta,$$
$$2\psi' = \gamma - \alpha + \delta, \qquad 2\phi' = \gamma + \alpha + \delta.$$

It may be noted that if a ray is refracted without axial aberration one of the angles α, γ, δ, vanishes. Thus if a ray meets the surface at its vertex, $\alpha = 0$; if it passes through the centre of curvature, $\delta = 0$; and if it passes through the non-coincident aplanatic points, $\gamma = 0$. This suggests that it may be possible to express the aberrations in terms of α, γ and δ.



In the figure let XP be an incident ray meeting the axis in X and the refracting surface in P. Suppose that A, C, and L are the vertex, the centre of curvature,

* γ is the angle made with the axis by the line joining the centre of curvature to the points of intersection of the rays with the aplanatic surfaces.

and the aplanatic point on the axis for incident rays. Let positive signs be used for lengths measured in the direction of the incident light, so that

$$AX = x,\ AC = r,\ CL = r\mu'/\mu.$$

From the triangle XPC

$$(x - r)\sin\psi = -r\sin\phi \qquad \ldots.(3),$$

and thus from the corresponding relation for the refracted ray and from the law of refraction

$$\mu\,(x - r)\sin\psi = \mu'\,(x' - r)\sin\psi',$$

or if $\mathcal{M}$ denote the sine ratio

$$\mathcal{M} = \frac{\mu\sin\psi}{\mu'\sin\psi'} \qquad \ldots.(4),$$

this equation leads to

$$x' - r = \mathcal{M}\,(x - r) \qquad \ldots.(5).$$

Equation (3) may be rewritten in the form

$$x\sin\psi = -2r\sin\tfrac{1}{2}\alpha\cos\tfrac{1}{2}(\gamma - \delta),$$

and therefore from this and the corresponding equation for the refracted ray

$$\frac{x'\sin\psi'}{x\sin\psi} = \frac{\cos\tfrac{1}{2}(\gamma + \delta)}{\cos\tfrac{1}{2}(\gamma - \delta)},$$

or from (4)

$$\frac{x'\mu}{x\mu'} = \mathcal{M}\,\frac{\cos\tfrac{1}{2}(\gamma + \delta)}{\cos\tfrac{1}{2}(\gamma - \delta)} \qquad \ldots.(6).$$

The two equations (5) and (6) give the exact equation of refraction

$$\frac{\mu'\,(1 - \eta)}{x'} - \frac{\mu\,(1 + \eta)}{x} = \frac{\mu' - \mu - (\mu' + \mu)\,\eta}{r} \qquad \ldots.(7),$$

where $\eta = \tan\tfrac{1}{2}\gamma\tan\tfrac{1}{2}\delta$. An alternative form is obtained by the retention of $\mathcal{M}$, giving the two equations

$$\left.\begin{aligned} xR\,\{\mu' - \mu - (\mu' + \mu)\,\eta\} &= -\mu\,(1 + \eta)\,(1 - 1/\mathcal{M}) \\ x'R\,\{\mu' - \mu - (\mu' + \mu)\,\eta\} &= \mu'\,(1 - \eta)\,(1 - \mathcal{M}) \end{aligned}\right\} \qquad \ldots.(8).$$

It will be noted that these equations differ from the paraxial equations only in the substitution of corrected refractive indices $\mu\,(1 + \eta)$ and $\mu'\,(1 - \eta)$ in place of the real indices μ and μ'. The utility of such equations lies in the fact that since η is a small correcting factor, it need only be known roughly to yield very accurate values of the coordinates of the refracted ray. The weak point of the equations is that the value of the η for a subsequent surface is not given by a simple formula in terms of that of the previous surface. The equations thus fail to give a simple method of tracing rays automatically through a series of surfaces.

The chief difficulty about the use of these equations in forecasting the magnitude of the aberrations lies in judging the magnitude of $\tan\tfrac{1}{2}\gamma$. The angle γ, unlike α and δ, is not readily visualised. It therefore seems desirable to obtain further equations in which the correcting factor is independent of γ.

To obtain such an equation refer the points of intersection of the ray with the axis to the aplanatic points.

Equation (3) gives

$$\begin{aligned}(x - r - r\mu'/\mu)\sin\psi &= -r(\sin\phi + \mu'\sin\psi/\mu)\\ &= -r(\sin\phi' + \sin\psi)\,\mu'/\mu\\ &= -2r\sin\tfrac{1}{2}\gamma\cos\tfrac{1}{2}(\alpha+\delta)\,\mu'/\mu,\end{aligned}$$

and therefore

$$\frac{(x' - r - r\mu/\mu')\sin\psi'}{(x - r - r\mu'/\mu)\sin\psi} = \frac{\mu^2\cos\tfrac{1}{2}(\alpha-\delta)}{\mu'^2\cos\tfrac{1}{2}(\alpha+\delta)},$$

or from (4)

$$\frac{\mu'x' - r(\mu+\mu')}{\mu x - r(\mu+\mu')} = \mathcal{M}\cdot\frac{1+\epsilon}{1-\epsilon} \qquad \ldots.(9),$$

where $\epsilon = \tan\frac{1}{2}\alpha\tan\frac{1}{2}\delta$. Equations (5) and (9) may be combined to yield the equations

$$\left.\begin{aligned}xR\{\mu' - \mu - (\mu'+\mu)\epsilon\} &= -\mu - (2\mu'+\mu)\epsilon + \mu(1-\epsilon)/\mathcal{M}\\ x'R\{\mu' - \mu - (\mu'+\mu)\epsilon\} &= \mu' - (\mu'+2\mu)\epsilon - \mu'(1+\epsilon)\mathcal{M}\end{aligned}\right\}\ldots.(10),$$

the result of eliminating $\mathcal{M}$ being

$$\frac{2(\mu'+\mu)\epsilon}{xx'} + \frac{\mu' - (\mu'+2\mu)\epsilon}{x'r} - \frac{\mu + (2\mu'+\mu)\epsilon}{xr} - \frac{\mu' - \mu - (\mu'+\mu)}{r^2} = 0\ldots.(11),$$

which is less simple than (7). There is however the compensating advantage that the magnitude of ϵ can often be written down with considerable precision. If the centre of curvature is taken as a reference point the equations take a simpler form, as is readily seen by rewriting (10) in the form

$$\left.\begin{aligned}(xR-1)\{\mu' - \mu - (\mu'+\mu)\epsilon\} &= -\mu'(1+\epsilon) + \mu(1-\epsilon)/\mathcal{M}\\ (x'R-1)\{\mu' - \mu - (\mu'+\mu)\epsilon\} &= \mu(1-\epsilon) - \mu'(1+\epsilon)\mathcal{M}\end{aligned}\right\}\ldots.(12),$$

leading to

$$\frac{\mu(1-\epsilon)}{x'-r} - \frac{\mu'(1+\epsilon)}{x-r} - \frac{\mu' - \mu - (\mu'+\mu)\epsilon}{r} = 0 \qquad \ldots.(13),$$

where the fourth term is absent. Similarly reference to the aplanatic points gives

$$\frac{\mu^2(1+\epsilon)}{\mu'x' - (\mu'+\mu)r} - \frac{\mu'^2(1-\epsilon)}{\mu x - (\mu'+\mu)r} - \frac{\mu' - \mu - (\mu'+\mu)\epsilon}{r} = 0.$$

The equations which have been reached involving ϵ and η have the common feature that if a plus sign is associated with these quantities for the incident ray, a minus sign is associated with them in the corresponding expressions for the refracted ray. This is evidently due to the difference in the sign of δ in the expressions for the fundamental angles in these two cases. The angles α and γ on the other hand retain the same signs for both incident and refracted rays. It is thus to be expected that the third form of the equation of refraction involving e, where $e = \tan\frac{1}{2}\alpha\tan\frac{1}{2}\gamma$, will be of a somewhat different character from the two expressions already determined. The equations are

$$\left.\begin{aligned}xR(\mu'-\mu)(1+e) &= -\mu + (2\mu'-\mu)e + \mu(1-e)/\mathcal{M}\\ x'R(\mu'-\mu)(1+e) &= \mu' + (\mu'-2\mu)e - \mu'(1-e)\mathcal{M}\end{aligned}\right\}\quad\ldots.(14),$$

or as a single equation

$$\frac{2(\mu'-\mu)e}{xx'} - \frac{\mu' + (\mu'-2\mu)e}{x'r} + \frac{\mu - (2\mu'-\mu)e}{xr} + \frac{(\mu'-\mu)(1+e)}{r^2} = 0\ldots.(15),$$

which may be compared with (10) and (11). When the centre of curvature is taken as the reference point equations (14) may be written

$$\left.\begin{aligned}(xR-1)(\mu'-\mu)(1+e) &= -(\mu'-\mu/\mathcal{M})(1-e)\\ (x'R-1)(\mu'-\mu)(1+e) &= (\mu-\mu'\mathcal{M})(1-e)\end{aligned}\right\} \quad \ldots.(16),$$

and therefore

$$\frac{\mu}{x'-r}-\frac{\mu'}{x-r}-\frac{\mu'-\mu}{r}\cdot\frac{1+e}{1-e}=0 \quad \ldots.(17),$$

which may be considered as an equation giving a shortened equivalent radius of curvature for oblique rays. Perhaps the simplest method of verifying these equations is to note that (16) involves (5) and that (17) satisfies (3) and the corresponding equation for the refracted ray.

It will be noted that of the three forms of the equation of refraction from which $\mathcal{M}$ has been eliminated, all reduce to three terms when the reference point is the centre of curvature and in addition the equation in η takes this simple form when the vertex is the reference point, the equation in ϵ when the aplanatic points are used for reference, while the equation in e does not reduce to three terms in either of these cases.

All the preceding formulae are examples of relations found by the elimination of some or all of the three quantities $\tan\frac{1}{2}\alpha$, $\tan\frac{1}{2}\gamma$, $\tan\frac{1}{2}\delta$ from four equations which express in terms of these tangents the law of refraction, the definition of $\mathcal{M}$, the relation between x and r, and that between x' and r. These relations may be exhibited in tabular form as shown below by inserting coefficients under the tangents to which they relate.

Relation	$\tan\frac{1}{2}\alpha$	$\tan\frac{1}{2}\gamma$	$\tan\frac{1}{2}\delta$	$\tan\frac{1}{2}\alpha\tan\frac{1}{2}\gamma\tan\frac{1}{2}\delta$	
Refraction	$\mu'-\mu$	$\mu'-\mu$	$\mu'+\mu$	$-(\mu'+\mu)$	....(18)
Magnification	$-(\mu'\mathcal{M}-\mu)$	$\mu'\mathcal{M}-\mu$	$\mu'\mathcal{M}+\mu$	$\mu'\mathcal{M}+\mu$	....(19)
Object point	$xR-2$	$-xR$	xR	$xR-2$	....(20)
Image point	$-(x'R-2)$	$x'R$	$x'R$	$x'R-2$	....(21)

For example the elimination of all the tangents gives equation (5); and the first of equations (8) is found from (18), (19), and (20) by substituting $\eta\tan\frac{1}{2}\alpha$ for the product of the three tangents and including this with the other terms in $\tan\frac{1}{2}\alpha$ before eliminating the three tangents.

The combination of any two of the equations (18) to (21) will enable the ratio of any two tangents, and consequently the ratio of any two of the three quantities ϵ, η, e, to be expressed in terms of one of them. Thus, noting that the headings of the four columns may be replaced by η^{-1}, ϵ^{-1}, e^{-1}, and 1 respectively, if it were desired to express η and e in terms of ϵ, μ, μ', and $\mathcal{M}$, equations (18) and (19) give

$$\begin{aligned}&\eta\{\mu\mu'(\mathcal{M}-1)+(\mu'+\mu)(\mu'\mathcal{M}+\mu)\epsilon\}\\ &\quad=-e\{(\mu'-\mu)(\mu'\mathcal{M}-\mu)-\mu\mu'(\mathcal{M}-1)\epsilon\}\\ &\quad=\epsilon(\mu'^2\mathcal{M}-\mu^2).\end{aligned}$$

The substitution of these values of η and e in equations (8) and (14) gives (10).

Such equations however are chiefly of use in obtaining the value of the quantity it is convenient to use when the value of another is better known. It will usually be unnecessary to take any account of the term involving a product of the small quantities, the approximate relations

$$\eta\mu\mu'(\mathcal{M}-1) = -e(\mu'-\mu)(\mu'\mathcal{M}-\mu) = \epsilon(\mu'^2\mathcal{M}-\mu^2)^* \quad \ldots(22)$$

being nearly always sufficiently exact. These equations show that e will usually be much larger than either η or ϵ. If the calculations are made from one of the equations (7), (11), or (15), or from the alternative forms referred to the centre of curvature, the approximate relations

$$\eta\mu' xR = -e(\mu'-\mu)(xR-1) = \epsilon(\mu xR-\mu'-\mu)$$

will usually be found more convenient.

When a plane surface is encountered only equation (7) of the new formulae is in a convenient form for use. The angle α becomes zero, and ϵ and e must be replaced by other quantities involving y, the distance from the axis of the point of refraction. The modified equations may be derived from the subjoined table.

Relation	y	$\tan\frac{1}{2}\gamma$	$\tan\frac{1}{2}\delta$	$y\tan\frac{1}{2}\gamma\tan\frac{1}{2}\delta$	
Refraction	0	$\mu'-\mu$	$\mu'+\mu$	0	....(23)
Magnification	0	$\mu'\mathcal{M}-\mu$	$\mu'\mathcal{M}+\mu$	0	....(24)
Object point	1	x	$-x$	1	....(25)
Image point	1	x'	x'	-1	....(26)

Comparison of (23) and (24) shows that $\mathcal{M}=1$, as is otherwise evident from the identity of ϕ and ψ.

When the correcting factors are known with sufficient accuracy for a series of surfaces, a slight modification of the ordinary paraxial formulae enables the position of the emergent ray to be found quickly. Suppose for instance that the value of η for each surface is known. Use suffixes to distinguish the surfaces. Let H_λ denote

$$\{\mu_\lambda - \mu_{\lambda-1} - (\mu_\lambda+\mu_{\lambda-1})\eta_\lambda\}R_\lambda.$$

Then the formulae

$$\left.\begin{aligned} H_{1,\lambda} &= (1+\eta_\lambda)H_{1,\lambda-1} + H_\lambda\frac{\partial H_{1,\lambda}}{\partial H_\lambda} \\ \frac{\partial H_{1,\lambda+1}}{\partial H_{\lambda+1}} &= (1-\eta_\lambda)\frac{\partial H_{1,\lambda}}{\partial H_\lambda} - \frac{t_\lambda}{\mu_\lambda}H_{1,\lambda} \end{aligned}\right\} \quad \ldots(27)$$

if applied to each surface in succession give $H_{1,n}$ and $\frac{\partial H_{1,n}}{\partial H_n}$. The corresponding reversed formulae are

$$\left.\begin{aligned} H_{\lambda,n} &= (1-\eta_\lambda)H_{\lambda+1,n} + H_\lambda\frac{\partial H_{\lambda,n}}{\partial H_\lambda} \\ \frac{\partial H_{\lambda-1,n}}{\partial H_{\lambda-1}} &= (1+\eta_\lambda)\frac{\partial H_{\lambda,n}}{\partial H_\lambda} - \frac{t_{\lambda-1}}{\mu_{\lambda-1}}H_{\lambda,n} \end{aligned}\right\} \quad \ldots(28),$$

* The exact relations may be written in the form

$$\frac{\eta\mu\mu'(\mathcal{M}-1)}{1+e\eta/\epsilon} = -\frac{e(\mu'-\mu)(\mu'\mathcal{M}-\mu)}{1-e^2} = \frac{\epsilon(\mu'^2\mathcal{M}-\mu^2)}{1+e\epsilon/\eta}.$$

and both should give the same value for $H_{1,n}$. The coordinates of the incident and refracted rays referred to the vertices of the external surfaces are

$$\left.\begin{aligned} x_1 H_{1,n} &= -\mu_0 (1+\eta_1) \frac{\partial H_{1,n}}{\partial H_1} + \mu_0 (1+\eta_1)(1+\eta_2)\,..\,(1+\eta_n)/\mathcal{M} \\ x'_n H_{1,n} &= \mu_n (1-\eta_n) \frac{\partial H_{1,n}}{\partial H_n} - \mu_n (1-\eta_1)(1-\eta_2)\,..\,(1-\eta_n)\,\mathcal{M} \end{aligned}\right\} \ldots\ldots(29),$$

where $\mathcal{M} = \dfrac{\mu_0 \sin \psi_0}{\mu_n \sin \psi_n}$. In this form there is a check on the accuracy of numerical work, but this may be dispensed with if desired in exactly the same way as is frequently done in calculating paraxial rays arising from a particular object point.

The corresponding formulae for a series of equations where ϵ or e is known instead of η may be written down by analogy with equations (27), (28), and (29) if it is noted that equations (10) may be written

$$xR\{\mu' - \mu - (\mu' + \mu)\epsilon\} + 2(\mu' + \mu)\epsilon = -\mu(1-\epsilon)(1 - 1/\mathcal{M}),$$
$$x'R\{\mu' - \mu - (\mu' + \mu)\epsilon\} + 2(\mu' + \mu)\epsilon = \mu'(1+\epsilon)(1 - \mathcal{M}),$$

showing that the equivalent unit point is moved from the vertex through a distance

$$\frac{2(\mu' + \mu)\epsilon r}{\mu' - \mu - (\mu' + \mu)\epsilon}$$

in a direction opposite to that in which the light traverses the system, and that equations (14) may be put into the form

$$xR(\mu' - \mu)(1+e) - 2(\mu' - \mu)e = -\mu(1-e)(1 - 1/\mathcal{M}),$$
$$x'R(\mu' - \mu)(1+e) - 2(\mu' - \mu)e = \mu'(1-e)(1 - \mathcal{M}),$$

with a displacement of the equivalent unit point equal to

$$\frac{2er}{1+e}$$

in the same direction as that followed by the light. In calculations with the ϵ formulae it follows that t_λ is to be replaced by

$$t_\lambda + \frac{2(\mu_\lambda + \mu_{\lambda-1})\epsilon_\lambda r_\lambda}{\mu_\lambda - \mu_{\lambda-1} - (\mu_\lambda + \mu_{\lambda-1})\epsilon_\lambda} - \frac{2(\mu_{\lambda+1} + \mu_\lambda)\epsilon_{\lambda+1} r_{\lambda+1}}{\mu_{\lambda+1} - \mu_\lambda - (\mu_{\lambda+1} + \mu_\lambda)\epsilon_{\lambda+1}},$$

and in the e formulae by $\quad t_\lambda - \dfrac{2e_\lambda r_\lambda}{1+e_\lambda} + \dfrac{2e_{\lambda+1} r_{\lambda+1}}{1+e_{\lambda+1}}$.

In these equivalent thickness formulae it will often be permissible to neglect the e's and ϵ's in the denominators. It is worthy of note that there is a displacement of the equivalent unit point from the present point of view even when refraction occurs at a flat surface, since in such cases ϵr and er in general are finite quantities as has been pointed out earlier.

The accuracy necessary in the quantities η, ϵ, and e can be estimated in terms of the accuracy required in the coordinates of the refracted ray by differentiating the foregoing equations. A limit to the necessary accuracy is however furnished by the consideration that η corresponds to a correction to the refractive index of the glass, and that nothing will consequently be gained by estimating it to an accuracy

greater than that corresponding to the variations from perfect homogeneity in a single piece of glass. In a good piece of glass the refractive index may easily vary over a few units in the fifth decimal place, and in many portions of instruments glass may be employed which exhibits variations in the fourth decimal place. For such purposes as the new equations are adapted to, it appears that η need not be known in absolute value more closely than about ·00002 or ·00003 while in many cases ·0002 or ·0003 will be reasonable limits. The corresponding absolute errors permissible in ϵ and e are given by equations (22). To convert these errors into relative values the order of magnitude of the correcting factors is required. An examination of the constructional data of photographic objectives given in von Rohr's *Theorie des photographischen Objektivs*, shows that in optical systems of this class, in which the conditions of refraction are often of an extreme character, the value of ϵ rarely exceeds ·02, values very much less than this being the rule. It may therefore be expected that the angles α, γ, δ should be known within an accuracy of from ·1 to 1 per cent.

A lens frequently has its curvature determined by the condition that a given ray of light is to be deviated through a known angle at a known distance from the axis of the system. In such cases one of the correcting factors will be known to a much higher order of accuracy than is necessary for these equations. The small relative accuracy required in the values of the angles makes it possible to use the equations in other cases when the values of the angles can be suitably estimated. It will often occur, particularly in compact systems or in combinations of separate systems each of which is approximately corrected for axial aberrations, that sufficiently exact values for these angles can be derived from paraxial calculations*. The coordinates found for the emergent ray in such a case differ from those given by expressions for first order aberrations, because in the latter type of calculation the products of the corrections arising at different surfaces are neglected, but in calculations according to the new equations these are all retained. The accuracy given in such calculations evidently corresponds to that obtained in trigonometrical ray tracing where the number of figures retained in the sine of an angle is insufficient to be affected by the fifth power of the angle. A number of illustrative calculations are appended to this note, and for the sake of comparison the values derived from tracing rays through the system are added.

The new equations suggest an interesting speculation. If a system of spherical surfaces is free from axial aberrations of all orders, the final x' is given in terms of the initial x by the paraxial formulae and also by any of the generalised formulae (7), (11) or (15) no matter what the values of the individual η's, ϵ's, or e's may be. It might thus be thought that a necessary, but not a sufficient, condition for freedom from aberrations of all orders would be found by supposing that the relation between the final x' and the initial x derived by putting the η's, ϵ's, or e's equal to zero (the paraxial conditions) would agree with that derived by making these quantities large (corresponding to large aberrations at the individual surfaces).

* Other methods, capable of greater accuracy, are available, but these need not be discussed here.

In other words, a necessary condition might be derived by equating the coefficients of the correcting factors to zero at each surface and solving the resulting chain of equations in parallel with the paraxial equations. It is however easy to see that in the case of the η and ϵ equations the result given by such a process is absurd. For the new η equation would be

$$\frac{\mu'}{x'} + \frac{\mu}{x} = \frac{\mu' + \mu}{r},$$

and the new ϵ equation could be written in the form

$$\frac{\mu'}{x' - 2r} + \frac{\mu}{x - 2r} = \frac{\mu' + \mu}{-r}.$$

Thus the η equation corresponds to the refraction of the paraxial rays at the vertex of the surface according to the usual laws except that the incident and refracted rays lie on the same side of the normal instead of on opposite sides of it. The same modified law of refraction holds for the ϵ equation, but the refraction here takes place at the opposite end of the diameter of the sphere. Now it is necessary for no difference to be made in the refraction by the interposition of an infinitely thin third medium between the two media considered. If this is done with this modified law of refraction the law is changed to the ordinary one. Calculations with these equations can thus carry no significance.

This objection does not hold in the case of the equations in e, as is evident from inspection of equation (17). The condition derived from this equation cannot however be necessary since it would not necessarily be satisfied by a train of surfaces in which successive aplanatic points or vertices coincided. It appears possible that some meaning may be given to calculations made with this further equation, but the subject needs further investigation. The foregoing considerations seem to point to the conclusion that freedom from spherical aberration of all orders in a system of spherical refracting surfaces bounding homogeneous media is, in all probability, only possible if there is freedom from aberrations at each individual refraction.

APPENDIX

A series of cemented objectives of increasing relative aperture may be taken to illustrate the application of the foregoing formulae. The crown component in each case is equiconvex, and the flint plano-concave. Other particulars are as follows:

Objective	No. 1	No. 2	No. 3	No. 4
Crown thickness ...	1·0	1·0	1·0	1·0
Flint thickness... ...	0·5	0·5	0·5	0·5
Full aperture	8·0	5·0	3·0	2·0
Radius of curvature ...	16·25	6·5	2·5	1·25
Crown index	1·52	1·52	1·5	1·5
Flint index	1·6188	1·6188	1·62	1·62
Paraxial power ...	·026048	·0656	·1584	·3296
Aperture/focal length ...	·208384	·328	·4752	·6592
Approximate *f* number	5	3	2	1·5

Paraxial rays incident on the crown lens parallel to the axis are brought to a focus at the following distances from the plane surface of the flint lens:

37·2736 14·1327 5·1627 1·9163

Marginal rays incident on the crown lens parallel to the axis are actually refracted through the systems in accordance with the following particulars:

$\tan \frac{1}{2}a_1$	·125	·2	·3333	·5
$\tan \frac{1}{2}\gamma_1$	·0815	·1286	·2087	·2889
$\tan \frac{1}{2}\delta_1$	−·0431	−·0696	−·1165	−·1844
$\tan \frac{1}{2}a_2$	−·125	−·2	−·3333	−·5
$\tan \frac{1}{2}\gamma_2$	−·2025	−·3297	−·5612	−·8972
$\tan \frac{1}{2}\delta_2$	·0106	·0179	·0423	·0975
y_2	3·9350	2·3964	1·3515	·8279
$\tan \frac{1}{2}\gamma_3$	−·0852	−·1363	−·1963	−·2283
$\tan \frac{1}{2}\delta_3$	−·0201	−·0322	−·0464	−·0540
x_3'	37·2763	14·1583	5·5180	2·8965
$-\sin \psi_3/y_1$	·026245	·066752	·158600	·274811

The discrepancy between the figures of the last line and the paraxial power gives the departure from the sine condition, and that between the figures of the penultimate line and the position of the paraxial focus the longitudinal spherical aberration. In the later members of the series the angles of incidence and refraction are decidedly large, as is shown by the following table which gives these angles to the nearest degree.

ϕ_1	14	23	37	53
ϕ_1'	9	15	24	32
ϕ_2	−19	−31	−50	−74
ϕ_2'	−18	−29	−45	−63
ϕ_3	− 4	− 6	− 8	−10
ϕ_3'	− 6	−10	−14	−16

With such large angles of refraction it is hardly to be expected that approximate values of the correcting factors based upon the paraxial foci will give very good results, but it will be of interest to see the magnitude of the resulting errors in these factors. The values of $\tan \frac{1}{2}a_1$ and $\tan \frac{1}{2}a_2$ can be written down exactly without calculation as the reciprocals of the full aperture*. An approximate value of y_3 is found by joining the point of intersection of the two spherical surfaces with the paraxial focus after the ray has passed through these two surfaces. The assumption that these marginal rays are refracted through the paraxial foci gives the following approximate values:

$\tan \frac{1}{2}a_1$	·125	·2	·3333	·5
$\tan \frac{1}{2}\gamma_1$	·0822	·1316	·2222	·3333
$\tan \frac{1}{2}\delta_1$	−·0428	−·0684	−·1111	−·1667
$\tan \frac{1}{2}a_2$	−·125	−·2	−·3333	−·5
$\tan \frac{1}{2}\gamma_2$	−·2021	−·3278	−·5556	−·9007
$\tan \frac{1}{2}\delta_2$	·0103	·0166	·0342	·0539
y_3	3·9348	2·3953	1·3398	·7563
$\tan \frac{1}{2}\gamma_3$	−·0854	−·1371	−·2099	−·3192
$\tan \frac{1}{2}\delta_3$	−·0202	−·0324	−·0497	−·0755

The magnitude of the resulting errors in η and e expressed as a percentage of the correct values are approximately

$100\,\Delta\eta_1$	$\frac{1}{4}$	$\frac{1}{2}$	$1\frac{1}{2}$	4
$100\,\Delta\eta_2$	−3	−7	−20	−45
$100\,\Delta\eta_3$	$\frac{1}{4}$	1	14	95
$100\,\Delta e_1$	1	2	7	15
$100\,\Delta e_2$	$-\frac{1}{4}$	$-\frac{1}{2}$	−1	$+\frac{1}{2}$
$100\,\Delta e_3$	$\frac{1}{4}$	$\frac{1}{2}$	6	28

The variations thus tend, even in the first member of the series, to be larger than the values indicated as desirable limits. The resulting errors in the values of x_3' and $-\sin\psi_3/y_1$ (or K) are

$\Delta_3 x_3'$	−·0241	−·0638	−·2833	−1·0330
$\Delta_e x_3'$	−·0077	−·0491	−·0771	+·1711
$\Delta x_3'$	−·0027	−·0256	−·3553	−·9802
$\Delta_3 K$	−·000088	−·000435	+·006212	+·093664
$\Delta_e K$	+·000007	+·000239	+·002220	−·009057
ΔK	−·000197	−·001152	−·000200	−·054789

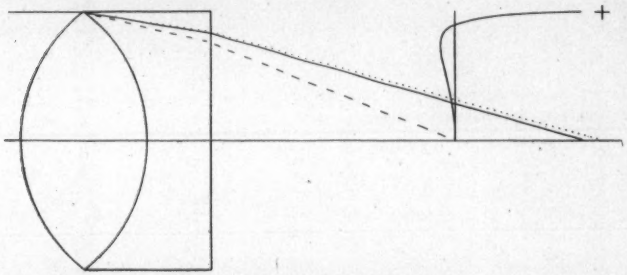
the values given in rows 3 and 6 being the errors involved by adopting paraxial values. It must be remembered that with objectives of this type the aberration is positive for the greater part of the aperture and becomes negative for a comparatively narrow outer zone. To this the character of the errors given by the η formulae is probably due. This feature is shown in the accompanying figure for the fourth objective, the spherical aberration determined from exact ray tracing

* If t is the axial thickness and y the maximum semi-aperture, $t = y(\tan \frac{1}{2}a_1 - \tan \frac{1}{2}a_2)$. If r_1 and r_2 are the radii, sufficiently exact values are given by

$$\tan \tfrac{1}{2}a_1 = -\frac{r_2}{r_1 - r_2} \cdot \frac{t}{y} + \frac{r_1 r_2 (r_1 + r_2)}{(r_1 - r_2)^3} \cdot \frac{t^3}{y^3},$$

$$\tan \tfrac{1}{2}a_2 = -\frac{r_1}{r_1 - r_2} \cdot \frac{t}{y} + \frac{r_1 r_2 (r_1 + r_2)}{(r_1 - r_2)^3} \cdot \frac{t^3}{y^3}.$$

being shown in the usual manner. The cross marks the value found for the marginal ray from these equations. The lens is shown in its proper position and the diagram makes clear the extreme character of this objective.



Objective No. 4 and its spherical aberration.

The intersection point for the marginal ray given by the e formulae is marked with a cross. The actual path of the marginal ray is marked by a continuous line. The line of dots marks the path given by the e equation to a first approximation. The line of dashes marks the path—through the same axial points as the paraxial rays—which was assumed in finding the values of the e's.

It is evident that the e formulae are the more satisfactory in use, and they are also the more convenient. If the results obtained by using the approximate values of the e's are used to obtain a second approximation to the true values, the resulting errors in the estimated aberrations will be much smaller. The results obtained for the paraxial calculation, first e approximation, second e approximation, and the exact values are, for x_3':

paraxial	37·2736	14·1327	5·1627	1·9163
first	37·2686	14·1091	5·4409	3·0676
second	37·2765	14·1590	5·5278	3·0454
exact	37·2763	14·1583	5·5180	2·8965

and for K:

paraxial	·026048	·065600	·15840	·32960
first	·026252	·066991	·16082	·26575
second	·026245	·066749	·15835	·26427
exact	·026245	·066752	·15860	·27481

It will be seen that the successive approximations converge very rapidly, and that in general they have the property, which is easily verified theoretically, of being alternately greater and less than the exact value. The apparent exceptions in the fourth objective are due to two effects which operate in different directions. A third approximation with the e formulae will give a good result. In the other three cases it is evident that a second approximation to e yields highly accurate results.

In the formulae which have been employed the fundamental quantity which determines the refraction is p where $p = 2er/(1 + e)$. The variation in this quantity is very small, as the following tables show:

	First surface p values			
first approx.	·33070	·33334	·34483	·35714
second approx.	·32777	·32583	·32389	·30822
exact value	·32779	·32600	·32523	·31557
	Second surface p values			
first approx.	−·80069	−·79991	−·78125	−·77626
second approx.	−·80251	−·80461	−·79039	−·78059
exact value	−·80247	−·80430	−·78799	−·77419
	Third surface p values			
first approx.	−·33599	−·32838	−·28116	−·24138
second approx.	−·33542	−·32663	−·26502	−·18352
exact value	−·33542	−·32663	−·26525	−·18898

These values suggest that it may prove possible to write down the values of p to a high order of accuracy for certain types of lens without special calculation. It would then be possible to trace marginal rays through a lens with sufficient accuracy with exactly the same amount of work as is required for paraxial rays, the same formulae being employed.

DISCUSSION

Mr P. F. Everitt said that it would be interesting from the practical point of view to know how the formulae would apply to the case of more than one refracting surface.

Instructor Commander T. Y. Baker asked if the author had considered the possibility of substituting arbitrary quantities instead of p in the equation

$$\frac{\mu'}{x'-p}-\frac{\mu}{x-p}=\frac{\mu'-\mu}{r-p}.$$

The value of the quantity in the first term would be determined by the constants of the incident ray and interesting results might be obtained when the values of the quantities in the other two terms were calculated.

The President (Prof. Cheshire) said he would have liked to have seen figures given for the case of systems having more moderate and practical apertures. It would be interesting to have comparison curves illustrating the results obtained from the author's formulae and those derived from accurate computation.

The author, in reply, stated that he did not propose the use of the formulae as a general substitute for methods of tracing rays. Before such a step could be taken much experience in the application of the new formulae would be necessary, particularly in proceeding from surface to surface. The new equations seemed to be particularly adapted for use in special cases, such as when $\tan\frac{1}{2}\alpha$, $\tan\frac{1}{2}\gamma$, or $\tan\frac{1}{2}\delta$ was known with fair accuracy. These conditions were realised in a number of cases, of which the numerical examples illustrated one. He thought that their

use in the calculation of eyepieces was promising. In any particular kind of case there were many different ways in which a solution could be obtained, and only experience could decide which method would give a result of the desired accuracy with the least trouble. In some systems a value of p based on paraxial results might give all that was required. In others the addition of tabulated corrections to the crude paraxially determined values of p might be the best solution. At present he was unable to give any opinion what course should be taken. The equations had been published for their own interest and in the hope that others would find them useful.

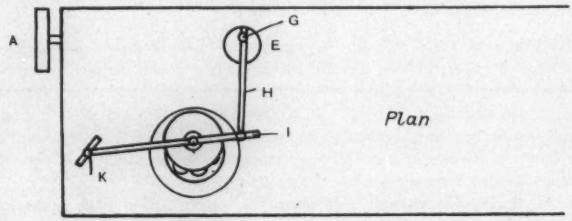
NOTES ON THE DESIGN AND MANUFACTURE OF BINOCULARS

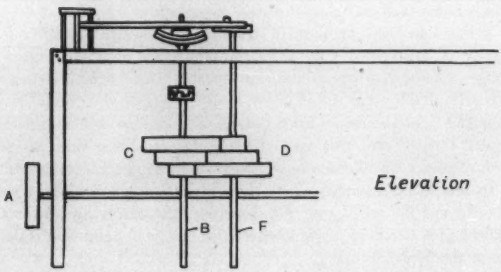
By H. S. RYLAND

Read and discussed, 14th November, 1918

Mr H. S. RYLAND gave a lecture on the Design and Manufacture of Binoculars. He discussed the faults which usually develop in binoculars from rough usage and ordinary wear, and suggested alterations in design so as to overcome these faults. He advocated the use of moulded discs for the lenses to save glass and to reduce the volume of material to be removed in grinding; in such a case he considered emery should be used throughout as an abrasive in preference to carborundum. In the metal work he considered pressed work should be more widely used, with expansion dies for cutting screw threads, particularly for lens cells and counter cells.

The design should be arranged for the use of a number of separate simple machines rather than a few complicated tools. Diagrams were shown illustrating the principles of a possible machine tool outfit. He advocated the use of a smoothing and polishing machine such as is shown in the following diagrams. The machine





is built entirely of angle iron, shafting, plummer blocks, and brass and steel tubing. There are no sliding surfaces and there is no friction drive. The main drive for a number of machines is from a back shaft *A*. A crossed belt from *A* drives the vertical spindle *B*. *E* is a slotted plate on the vertical shaft *F*, which is driven from *B*, a pair of cones *C*, *D* providing a range of relative speeds. A pin *G*, adjustable in the slot *E*, drives the arm *H*. This is connected by a pin to the arm *I*, which is pivoted at *K*, a pin in *I* controlling the movement of the tool. The machine is controlled by a pedal clutch on the pulley *B*. Within definite limits the relative speeds of *B* and *F* and the position and length of the stroke can be varied for figuring purposes. The arms *H* and *I* should be as light as possible; they may be made of wood when used on a small scale for eye-lenses. This machine can be built in various sizes for different classes of work.

Types of optical constructions were shown and it was suggested that a three-piece cemented objective appeared to give a more brilliant image than those of the usual two-piece type, while, owing to the flatter curves, it was probably but little more expensive to produce.

The restriction of manufacture to a few standardised designs was advocated, and in conclusion the lecturer considered the workshop testing of instruments.

DISCUSSION

The President (Prof. Cheshire) hoped that manufacturers would give the results of their experience on some of the important points raised. For example, the question whether die castings or sand castings should be used was a very important one. He had been told by one of the best makers that sand casting was essential for the bodies of binoculars in which the prisms rest in seats milled out of the body diaphragm and are secured by punching the metal near the edge of the seating and by a spring holding the prism down.

Mr H. A. Hughes thought that usually a longer period than 24 hours was required for the process of fine annealing.

Mr T. Smith communicated the following remarks:

Mr Ryland has asked my opinion of his proposal to use a resolving power test as a measure of definition. I consider that there are great difficulties in the way. Assume that a 30 mm. power 6 binocular is used. If the light is strong enough for the observer's pupil diameter to fall below 5 mm., the test object will have to be changed to suit the light intensity, since the eye and not the binocular limits the resolving power. Apart from this case the angular resolving limit as the rays emerge from the eyepiece is about one-third of a minute, decidedly less than the eye resolves. In testing instruments where these objections do not hold a resolving power test should not be relied on. For instance, the resolving power may be maintained right across the field of view where the marginal definition is decidedly less good than at the centre.

It is roughly correct to say that provided the aberrations do not pass limits which can be ascertained in any particular case the resolving power is unaffected.

Mr Ryland, in reply, stated that an annealing period of 48 hours was sufficient for the prisms and objectives of 6 × 30 binoculars, while for smaller work 24 hours was all that was necessary.

Die castings in several instances are being used for binocular bodies with complete success.

With the exception of the proposed definition test all of the methods described are in use.

EXHIBIT OF CAPTURED GERMAN OPTICAL INSTRUMENTS

By Lt.-Col. A. C. WILLIAMS, R.A.

Exhibited, 14th November, 1918

(1) and (2). Two small telescopes which are presumably for anti-aircraft work. They are marked "foot artillery," but the purpose for which they were used is not exactly known. The part that looks like a focussing gear is really a means of putting on deflection, by moving the graticule to the right or left. The optical particulars are as follows:

	Right-angle telescope	Prism Monocular telescope
Magnification	× 3	× 4
Field of view	11° 40′	9° 30′
Aperture of O.G.	·57	·56
Diameter of emergent pencil...	·19	·14
Definition	6° good; falls off rapidly towards edge with distortion; field not flat	Good over nearly whole field

(3) A small instrument which may be used as a director or dial sight. It is made of steel, can be set at angles of 1 millième, and has a collimator for laying with.

(4) A very strong and efficient ball and socket clamp. It appears, however, to be unnecessarily complicated.

EXHIBIT OF SAMPLES OF GLASS SHOWING FILMING

By W. H. WITHEY, B.A.

(From the National Physical Laboratory)

Exhibited, 14th November, 1918

The samples of optical glass exhibited were tested and classified in December, 1916, by exposing the broken surfaces to a moist atmosphere at 18° C. for one week and estimating by means of iodeosin the alkali liberated. Nos. 1, 2, 4, and 5 gave excellent results, whilst No. 3 was very distinctly inferior (H_5). Since December, 1916, the specimens of glass have been exposed to the weather on an open roof. The result of the test is that No. 3 is now distinctly the best, No. 4 comes next, No. 1 is almost as good, while No. 2, which previously gave results equal to No. 1, is seriously tarnished. Nos. 1 and 2 are of similar composition, as are Nos. 4 and 5.

The conclusion to be drawn is that the iodeosin test alone does not seem to be sufficient from the point of view of filming, and other tests seem to be desirable.

ALFRED SHERWOOD ESSLEMONT

OBITUARY NOTICE

ALFRED SHERWOOD ESSLEMONT, C.B.E.

We regret to announce the death on September 14th, 1918, of Mr Alfred Sherwood Esslemont, C.B.E., a Fellow of this Society, who at the time of his death was Controller of the Optical Munitions and Glassware Department, and also of the Potash Production Department, of the Ministry of Munitions.

When the Ministry of Munitions was created in 1915 Mr Esslemont joined the Department of Sir Eric Geddes, in which, with our President, Professor Cheshire, as a colleague, he assumed responsibility for the production of optical munitions of war. When it is remembered that prior to the war none of these munitions, with the exception of rangefinders and binoculars, had ever been made in England on a manufacturing scale, the difficulty of the task of organising the optical industry to meet the enormous demands suddenly made upon it will be realised. This difficulty was increased by the fact that before the war this country had very largely to depend upon foreign sources for its supply of optical glass, so that before adequate production was possible it was necessary very greatly to increase the output of this indispensable material. The success attending Mr Esslemont's efforts is now a matter of history. Optical munition works sprang up under the genius of his efforts, and the existing factories were extended and equipped in all directions. Most men would have been satisfied with the work involved in the supply of optical munitions, but later Mr Esslemont added to this the responsibility for the production and distribution of glassware generally, and later still that of potash production. Indeed there is little doubt that the work and anxiety entailed by the latter problem hastened his end. Mr Esslemont was preeminently the man for the hour. He combined in a rare degree the great qualities of far-seeing enterprise, indomitable doggedness, and courage. Difficulties served but to stimulate him to further and more strenuous effort. Under his inspiration and guidance the British optical industry rose magnificently to the tasks imposed upon it and thus contributed in no small degree to the glorious success which has attended our national efforts.

For some time before Mr Esslemont's death his friends recognised the seriousness of his illness and urged upon him the necessity for taking rest, but he had determined to fight to the last and go down, if it must be, with his flag flying. His end came with victory in sight. He sacrificed his life for his country just as surely as if he had fallen in the trenches at the Front.

REQUIESCAT IN PACE

ABSTRACTS

"On the Correction of Optical Surfaces." By A. A. MICHELSON, *Astrophysical Journal*, 47 (1918), 283.

The author points out that F. Twyman's interferometer method of correcting optical surfaces [*Phil. Mag.* 35 (1918), 49] cannot very well be applied to the case of mirrors and lenses of the size of astronomical telescopes, as it would involve the use of interferometers of at least equal dimensions. He describes a simple method of testing a concave mirror 6 inches in diameter and of 36 inches radius. Light from a slit, illuminated by a Nernst lamp, is allowed to fall on the mirror after reflection by a right-angled prism. The image of the slit is formed immediately above the prism and is viewed with a $\frac{1}{12}$-inch objective. Screens with rectangular apertures P, Q are placed immediately in front of the mirror, P being at the centre and Q at varying distances from the centre of the mirror. If the mirror is perfect, the central interference band produced corresponds with the position of the slit image. The distance between the centres of band and slit image in fractions of the fringe-width gives twice the error of the mirror, in light-waves, at the point of the mirror corresponding to the position of Q. These errors are determined for positions of Q 2, 3, 4, 5, 6, and 7 cms. distant from P on either side of a diameter and at intervals of 45° rotation of the mirror about its axis. The values obtained in two independent sets of measurements agreed to within ·01 of a light-wave.

A similar experiment was tried with a 5 inch achromatic lens by O. L. Petitdidier, $f = 12$ ft., backed by a plane mirror used for spectrographic work. On account of the secondary chromatic aberration the light used was approximately homogeneous, corresponding in wave-length with the yellow Hg line.

The method is applicable to the largest astronomical mirrors or lenses as an aid to the original figuring and the final corrections; the latter may be made by an auxiliary correcting plate placed between the objective and the focus.

Great care must be taken in the focal adjustment, and also in the selection and maintenance of the wave-length of the light employed in the case of an achromatic lens.

The same method, with modifications, may be applied to the correction of prisms and diffraction gratings; a single plane parallel plate may be employed. The interference system consists of a series of concentric circles. If the grating under examination and the auxiliary surfaces be perfect, the diameter of the circular fringes remains constant when the eye or the observing telescope is moved about in any direction.

Very interesting photographs illustrating the results obtained are given in the paper.

J. S. A.

"Knife-Edge Shadows." By RUSSELL W. PORTER, *Astrophysical Journal*, 47 (1918), 324.

One of the chief optical tests in determining the figure of the speculum in a reflecting telescope is the characteristic shadow which covers a paraboloidal mirror when a knife-edge intercepts the cone of light rays from its surface. The author has obtained interesting photographs of the phenomenon by placing a photographic plate behind the knife-edge

at the centre of curvature of a concave mirror. These photographs reveal imperfections of the glass which cannot be seen by the naked eye; they are therefore of great assistance in the final figuring of a mirror.

In one arrangement the author employs an artificial star, consisting of a small pin-hole backed by an acetylene flame, at the centre of curvature of the mirror. The knife-edge travels on a track parallel to the optical axis of the mirror and may also be adjusted in a transverse direction. In another arrangement the knife-edge and artificial star are at the focus. The mirror, of 10 inches diameter, had been converted into the Cassegrain form, the centre being bored out. A flat mirror is placed opposite and facing the mirror under examination at a distance of about one-half the focal length of the latter. Artificial star, knife-edge, and photographic plate are behind the parabolic mirror. The beam of light suffers five reflections, so that inequalities of the parabolic surface reach the plate exaggerated by double their true value. If the mirror under test were a perfect paraboloid the photograph obtained would be that of an absolutely flat disc, so that any variations from perfection in the surface can be detected.

In the case of the mirror tested by the author the photographs obtained show the effects of local polishing, the marks of the rose tool, small epicycloidal strokes of the polisher hitherto unsuspected, and even the accidental scratches on the mirror. The order of magnitude of the quantities dealt with is very small. The amount taken off the surface of the mirror near the centre, in passing from the sphere to the paraboloid, is something like one hundred-thousandth of an inch (·00025 mm.). The shadows of the depressions produced by the rose tool indicate a departure from the true paraboloid surface of only a few millionths of an inch.

The author draws attention to somewhat similar photographic records of optical surfaces of revolution obtained by J. Hartmann [*Z. für Instrumentenkunde*, 29 (1909), 217; *Astrophysical Journal*, 27 (1908), 237, 257]. In the Hartmann "focographs" the plate is focussed on the objective itself by the introduction of a camera lens, whereas the author's photographs are obtained directly, thus freeing the results from any errors due to the addition of another lens in the optical system.

J. S. A.

THE AITCHISON MEMORIAL SCHOLARSHIP

AT a meeting of the Aitchison Memorial Scholarship Committee held on November 3rd, 1918, the above Scholarship was awarded to Mr V. C. Milligen, 20, Aberdour Road, Goodmayes, Essex. The Scholarship is tenable at the Northampton Polytechnic Institute for two years, 1918–1920.

BRITISH SCIENTIFIC INSTRUMENT RESEARCH ASSOCIATION

THE following notice has been received from the Secretary of the above Association:

The BRITISH SCIENTIFIC INSTRUMENT RESEARCH ASSOCIATION, one of the earliest associations formed under the scheme of the Department of Scientific and Industrial Research, has secured premises at 26, Russell Square, W.C. 1, where offices and research laboratories will be equipped. The first Chairman of the Association was Mr A. S. Esslemont, whose recent lamented death has been a severe loss to the Association. The Council have elected Mr H. A. Colefax, K.C., as Chairman to fill the vacancy. The Vice-Chairman is Mr Conrad Beck, C.B.E., to whose energy and personal influence is largely due the successful formation of the Association. Almost all the leading Optical and Scientific Instrument manufacturers are members. The Department of Scientific and Industrial Research is represented by Major C. J. Stewart, Captain F. O. Creagh-Osborne, R.N., C.B., Mr S. W. Morrison, O.B.E., Colonel R. E. Home, D.S.O., R.A., and Mr Percy Ashley. The Council have recently co-opted as members of their body the Hon. Sir Charles A. Parsons, F.R.S., and Professor J. W. Nicholson, M.A., D.Sc., F.R.S. Sir Herbert Jackson, K.B.E., F.R.S., F.I.C., has been appointed Director of Research and Mr J. W. Williamson, B.Sc., Secretary of the Association.